Practice Exam 1
Chapter 1 and Chapter 2 (2.1-2.2)

Linear Functions:

Exercise 1: A city's population was 30,700 in the year 2010 and is growing by 850 people a year.

(a) Give a formula for the city's population, P, as a function of the number of years, t, since 2010.

(b) Whatis the population predicted to be in 2020?

(c) When is the population expected to reach 45,000?

Exercise 2: A company rents cars at \$40 a day and 15 cents a mile. Its competitor's cars are \$50 a day and 10 cents a mile.

(a) For each company, give a formula for the cost of renting a car for a day as a function of the distance traveled.

(b) How should you decide which company is cheaper?

Average and Relative Change:

Exercise 3: Which relative change is larger? An increase in class size from 5 to 10 or 30 to 50.
Exercise 4: Let $f(t)$ be the number of US billionaires in year t.

(a) Express the following statements in terms of f.

(i) In 1985 there were 13 US billionaires.

(ii) In 1990 there were 99 US billionaires.

(b) Find the average yearly increase in the number of US billionaires between 1985 and 1990. Express this using f.

(c) Assuming the yearly increase remains constant, find a formula predicting the number of US billionaires in year t.

Applications to Economics:

Exercise 5: An online seller of T-shirts pays \$500 to start up the website and \$6 per T-shirt, then sells the T-shirts for \$12 each. Find the profit function for the seller. How many T-shirts must he sell to break even?

Exercise 6: A new bus is worth \$100,000 in 2010 and depreciates linearly to \$20,000 in 2030.

(a) Find a formula for the value of the bus, V, as a function of time, t, in years since 2010.

(b) What is the value of the bus in 2015?

(c) Find and intrepret the vertical and horizontal intercepts of the graph of the function.

(d) What is the domain of the function?

Exercise 7: The demand curve of a product is given by $q = 120,000 - 500p$ and the supply curve is given by $q = 1000p$, where price is in dollars.

(a) At a price of \$100, what quantity are consumers willing to buy and what quantity are producers willing to supply? Will the market push prices up or down?

(b) Find the equilibrium price and quantity. Does your answer to part (a) support the observation that market forces tend to push prices closer to equilibrium price?

Exponential and Logarithm Functions:

Exercise 8: Find the value in 8 years of a \$10,000 investment at a rate of 3% compounded continuously.

Exercise 9: You make an investment today at an interest rate of 4% compounded continuously in which you want to have \$8,000 in 5 years. How much do you need to invest today?

Exercise 10: If \$12,000 is deposited in an account paying 8% interest per year, compounded annually, how long will it take for the balance to reach \$20,000?

Exercise 11: A person is to be paid \$2000 for work done over a year. Three payment options are to be considered. Option 1 is to pay the \$2000 in full now. Option 2 is to pay \$1000 now and \$1000 in a year. Option 3 is to pay the full \$2000 in a year. Assume an annual interest rate of 5% a year, compounded continuously. Without doing any calculations, which option is the best option financially for the worker? Explain your reasoning.

Exercise 12: Find the doubling time of a quantity that is increasing by 7% a year.

Exercise 13: A cup of coffee contains 100 mg of caffeine, which leaves the body at a continuous rate of 17% per hour.

(a) Write a formula for the amount, A mg, of caffeine in the body t hours after drinking a cup of coffee.

(b) Find the half-life of caffeine.

Exercise 14: A firm decides to increase output at a constant relative rate from its current level of 20,000 to 30,000 units during the next five years. Calculate the annual percent rate of increase required to achieve this growth.

Exercise 15: During a recession a firm's revenue declines continuously so that the revenue, R (in millions), in t years' time is given by $R = 5e^{-0.15t}$. After how many years will the revenue decline to 2.7 million?

Exercise 16: The population of the US was 281.4 million in 2000 and 308.7 million in 2010. Assuming exponential growth,

(a) In what year is the population expected to go over 350 million?

(b) What population is predicted for the 2020 census?

Composite Functions

Exercise 17: Let $f(x) = x^2$, $g(x) = 1/x$, $h(x) = \sqrt{x-4}$ and $\ell(x) = 3x + 2$.

(a) Find $f \circ \ell(x)$.

(b) Find $g(f(x))$.

(c) Find $h(g(1/8))$.

(d) Find $\ell(g(3))$.

(e) Find $g \circ h(x)$.

(f) Find $\ell(h(x+4))$.

Exercise 18: Use the variable u for the inside function to express each of the following as a composite function.

(a) $C = 12\ln(q^3 + 1)$

(b) $P = 16e^{-0.6t}$

(c) $y = (5t^2 - 2)^6$

Instantaneous Rate of Change and the Derivative Function

Exercise 19: Consider the function $f(x) = e^x$.

(a) Find the average velocity between $t = 0$ and $t = 0 + h$ if:

$$(i) \quad h = 0.1, \quad (ii) \quad h = 0.01, \quad (iii) \quad h = 0.001.$$

(b) Use your answers to part (a) to estimate the instantaneous velocity of the particle at time $t = 1$.

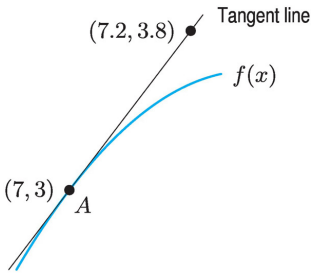
Exercise 20: Consider the function $f(x) = x^3$.

(a) Find the average velocity between $t = 2$ and $t = 2 + h$ if:

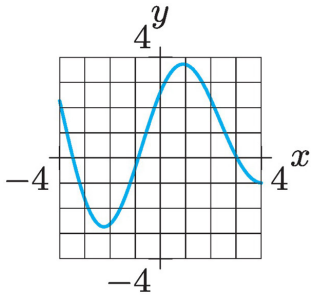
$$(i) \quad h = 0.1, \quad (ii) \quad h = 0.01, \quad (iii) \quad h = 0.001.$$

(b) Use your answers to part (a) to estimate the instantaneous velocity of the particle at time $t = 1$.

Exercise 21: Use the figure below to find $f(7)$ and $f'(7)$.



Exercise 22: Graph the derivative of



Exercise 23: Graph the derivative of